Algebra II Mathematics Content Standards

This discipline complements and expands the mathematical content and concepts of Algebra I and geometry. Students who master Algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

Note: The sample problems illustrate the standards and are written to help clarify them. Some problems are written in a form that can be used directly with students; others will need to be modified before they are used with students.

1.0 Students solve equations and inequalities involving absolute value.

Sketch the graph of each function.

$y = \left|\frac{1}{x}\right|$.

$y = -\frac{2}{3}|x - 2| - 5$.

2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.

Draw the region in the plane that is the solution set for the inequality $(x - 1)(x + 2y) > 0$.

3.0 Students are adept at operations on polynomials, including long division.

Divide $x^4 - 3x^2 + 3x$ by $x^2 + 2$, and write the answer in the form:

$$\text{polynomial} + \frac{\text{linear polynomial}}{x^2 + 2}.$$

4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

Factor $x^3 + 8$.

5.0 Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

6.0 Students add, subtract, multiply, and divide complex numbers.

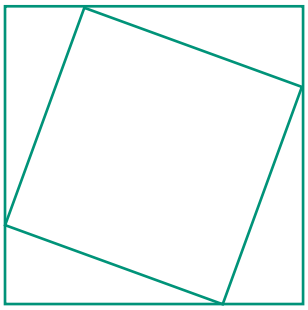
Write $\frac{1+i}{1-2i}$ in the form of $a + bi$, where a and b are real numbers.

7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator.

Simplify $\dfrac{\left(x^2 - x\right)^2}{x\left(x-1\right)^{-2}\left(x^2+3x-4\right)}$.

8.0 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system.

In the figure shown below, the area between the two squares is 11 square inches. The sum of the perimeters of the two squares is 44 inches. Find the length of a side of the larger square. (ICAS 1997, 12)



9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as *a, b,* and *c* vary in the equation $y = a(x - b)^2 + c$.

10.0 Students graph quadratic functions and determine the maxima, minima, and zeros of the function.

Find a quadratic function of x that has zeros at $x = -1$ and $x = 2$. Find a cubic equation of x that has zeros at $x = -1$ and $x = 2$ and nowhere else. (ICAS 1997, 7)

11.0 Students prove simple laws of logarithms.

11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Solve: $2^x = 5(13^{2x-5})$.

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11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step.

12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

Algebra II

The number of bacteria in a colony was growing exponentially. At 1 p.m. yesterday the number of bacteria was 100, and at 3 p.m. yesterday it was 4,000. How many bacteria were there in the colony at 6 p.m. yesterday? (TIMSS gr.12, K-13)

13.0 Students use the definition of logarithms to translate between logarithms in any base.

14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

1. Find the largest integer that is less than:

   $\log_{10}(1{,}256)$

   $\log_{10}(.029)$

2. $\frac{1}{2}\log_2 64 = ?$

15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true.

For positive numbers x and y, is the equation $\log_2 xy = \log_2 x \cdot \log_2 y$ always true, sometimes true, or never true?

If c is a real number, for what values of c is it true that $\frac{\sqrt{(c^2-1)^4}}{c+1} = c-1$?

16.0 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.

What is the graph of $x^2 + py^2 - 4x + 10y - 26 = 0$ when $p = 1$? When $p = 4$? When $p = -4$?

17.0 Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation.

Does the origin lie inside, outside, or on the geometric figure whose equation is $x^2 + y^2 - 10x + 10y - 1 = 0$? Explain your reasoning. (ICAS 1997, 11)

18.0 Students use fundamental counting principles to compute combinations and permutations.

19.0 Students use combinations and permutations to compute probabilities.

20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers.

What is the third term of $(2x - 1)^6$? What is the general term? What is a simplified expression for the sum?

21.0 Students apply the method of mathematical induction to prove general statements about the positive integers.

Use mathematical induction to prove that for any integer $n \geq 1$, $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series.

Find the sum of the arithmetic series: $13 + 16 + 19 + \ldots + 94$.

Find the sum of the geometric series:

$$\frac{3^5}{5^2} + \frac{3^6}{5^3} + \frac{3^7}{5^4} + \ldots + \frac{3^{32}}{5^{29}}.$$

23.0 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series.

24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

Which of the following functions are their own inverse functions? Use at least two different methods to answer this question and explain your methods:

$$f(x) = \frac{2}{x} \qquad g(x) = x^3 + 4 \qquad h(x) = \frac{2 + \ln(x)}{2 - \ln(x)} \qquad j(x) = \sqrt[3]{\frac{x^3 + 1}{x^3 - 1}}$$

(ICAS 1997, 13)

25.0 Students use properties from number systems to justify steps in combining and simplifying functions.